Production-Inventory strategy of deteriorating items for single-vendor and two-buyers.

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Abstract:

An aim of this article is to analyze a single-vendor, two buyers production-inventory policy for a deteriorating item under assumption of constant production and demand rate. A mathematical model is derived to compute total joint integrated cost of both the vendor and two-buyers. It is established that joint integrated cost is beneficial compared to the independent decisions made by the two stackers' viz. the vendor and the two-buyers.

Key-Words: Single-vendor, two-buyers, deterioration, production-inventory policy.

1. Introduction:

The researchers are engaged in analyzing optimal policies from buyer's point of view. These optimal policies may not be favorable to vendor. Banerjee (1986) developed a joint economic lot-size model for a single-buyer and single-vendor inventory system. Goyal (1988) generalized aforesaid model under the assumption of the lot-for-lot policy of the vendor. Ha and Kim (1997) formulated a mathematical model in which the inventory cost of vendor is derived through a discontinuous saw-tooth inventory-level system. Wee and Jong (1998) developed the manufacturer's integration between multi-parts and finished product with multi-lot-size for deteriorating items. For deteriorating inventory models in literature one can go through review articles by Raafat (1991), Shah and Shah (2000), Goyal and Giri (2001). Yang and Wee (2000) developed an integrated economic ordering policy of deteriorating items for a vendor and a buyer. Wu and Wee (2001) extended above concept by considering multiple lot size deliveries.

In this paper, a production-inventory system model of deteriorating items is developed for a single vendor and two-buyers. Numerically it is established that the integrated joint total cost is smaller than the total cost of a vendor and two buyers independently.

2. Assumptions and Notations:

The proposed model is derived under the following assumptions:

- An inventory system deals with single item.
- Single-vendor and two-buyers are considered.
- The production rate is finite and is greater than the sum of the demands of two buyers.
- Shortages are not allowed. Lead-time is zero or negligible.
- The deteriorated units can neither be replaced nor repaired during the cycle time.

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Notations are as follows:

θ_V : Deterioration rate of units in the vendor's inventory system.

θ_{B1} : Deterioration rate of units in the first buyer's inventory systems.

θ_{B2} : Deterioration rate of units in the second buyer's inventory systems.

Note: $\theta_V < \theta_{B1} < \theta_{B2}$

R_1 : Demand rate per year for first buyer.

R_2 : Demand rate per year for second buyer.

P : Production rate per year for vendor.

Note: $P > R_1 + R_2$

T_1 : Length of production time in each production cycle.

T_2 : Length of non-production time in each production cycle.

$T : (= T_1 + T_2)$ Length of one cycle.

$I_{v1}(t_1)$: Inventory level for vendor when $t_1 \in [0, T_1]$

$I_{v2}(t_2)$: Inventory level for vendor when $t_2 \in [0, T_2]$

n_1 : number of deliveries per cycle T for first buyer.

n_2 : number of deliveries per cycle T for second buyer.

$I_{B1}(t)$: Inventory level for first buyer when $t \in \left[0, \frac{T}{n_1}\right]$

$I_{B2}(t)$: Inventory level for second buyer when $t \in \left[0, \frac{T}{n_2}\right]$

$I_{mV}(t)$: Maximum inventory level for the vendor.

$I_{mB1}(t)$: Maximum inventory level for first buyer.

$I_{mB2}(t)$: Maximum inventory level for second buyer.

C_V : Unit production cost for vendor.

C_B : Unit purchase cost for both the buyers.

A_V : Ordering cost of each production cycle for vendor.

A_B : Ordering cost per order for a buyer.

h_V : Inventory holding cost per unit per time unit for vendor.

h_{B1} : Inventory holding cost per unit per time unit for first buyer.

h_{B2} : Inventory holding cost per unit per time unit for second buyer.

Note: $h_{B1} < h_{B2}$

K_V : Total cost of vendor per time unit.

K_B : Total cost of both the buyers per time unit.

K_J : The integrated cost of the vendor and both the buyers per time unit.

3. Mathematical model:

We analyze one cycle for the vendor and both the buyers. (Figs. 1 and 2)

The inventory for the vendor and the two buyers; B_1, B_2 at any instant of time t is governed by the following differential equations:

$$\frac{dI_{V1}(t_1)}{dt_1}+\theta_V I_{V1}(t_1)=P-(R_1+R_2), 0\le t_1\le T_1 \quad (1)$$

$$\frac{dI_{V2}(t_2)}{dt_2}+\theta_V I_{V2}(t_2)=-(R_1+R_2), 0\le t_2\le T_2 \quad (2)$$

$$\frac{dI_{B1}(t)}{dt}+\theta_{B1} I_{B1}(t)=-R_1, 0\le t\le \frac{T}{n_1}; \quad (3)$$

and $$\frac{dI_{B2}(t)}{dt}+\theta_{B2} I_{B2}(t)=-R_2, 0\le t\le \frac{T}{n_2} \quad (4)$$

with boundary conditions $I_{V1}(0)=0, I_{V2}(T_2)=0, I_{B1}\left(\frac{T}{n_1}\right)=0, I_{B2}\left(\frac{T}{n_2}\right)=0.$

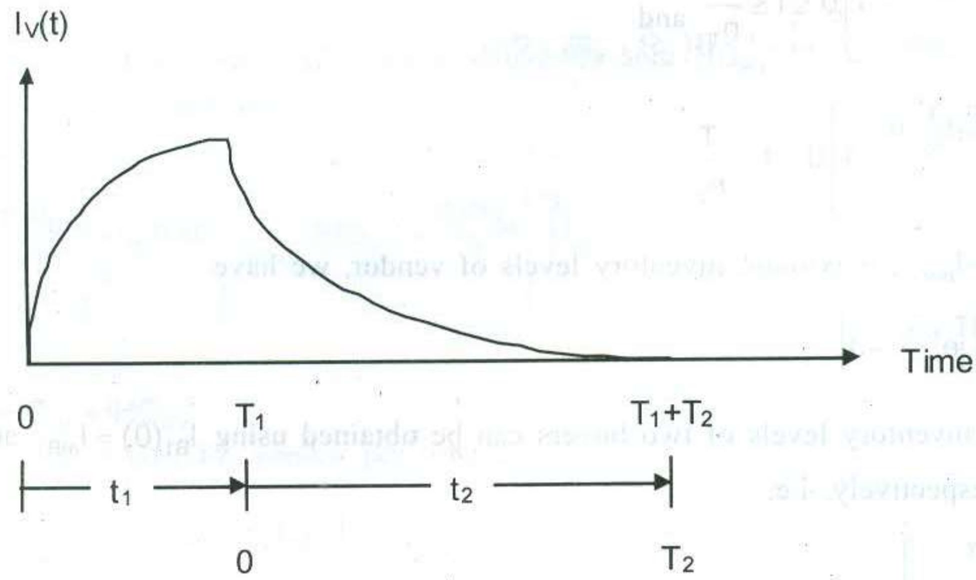

Fig.1 Production-inventory system for the vendor.

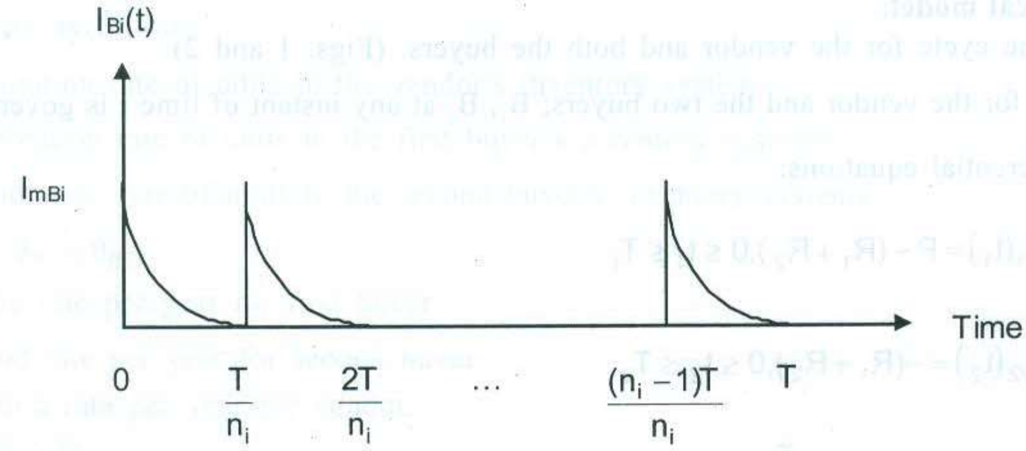

Fig.2 Inventory system for buyer i; i=1,2.

The solution of the differential equations (1) - (4) are :

$$I_{V1}(t_1)=\frac{P-(R_1+R_2)}{\theta_V}\left[1-e^{-\theta_V t_1}\right],0\le t_1\le T_1$$

$$I_{V2}(t_2)=\frac{(R_1+R_2)}{\theta_V}\left[e^{\theta_V(T_2-t_2)}-1\right],0\le t_2\le T_2$$

$$I_{B1}(t)=\frac{R_1}{\theta_{B1}}\left[e^{\theta_{B1}(\frac{T}{n_1}-t)}-1\right],0\le t\le\frac{T}{n_1}\text{ and}$$

$$I_{B2}(t)=\frac{R_2}{\theta_{B2}}\left[e^{\theta_{B2}(\frac{T}{n_2}-t)}-1\right],0\le t\le\frac{T}{n_2}.$$

Using $I_{V2}(0)=I_{mV}$; maximum inventory levels of vendor, we have

$$I_{mV}=\frac{(R_1+R_2)}{\theta_V}\left[e^{\theta_V T_2}-1\right] \tag{5}$$

The maximum inventory levels of two buyers can be obtained using $I_{B1}(0)=I_{mB1}$ and $I_{B2}(0)=I_{mB2}$ respectively. i.e.

$$I_{mB1}=\frac{R_1}{\theta_{B1}}\left[e^{\frac{\theta_{B1}T}{n_1}}-1\right]. \tag{6}$$

$$I_{mB2}=\frac{R_2}{\theta_{B2}}\left[e^{\frac{\theta_{B2}T}{n_2}}-1\right]. \tag{7}$$

Since, inventory at any instant of time t; $I_V(t)$ is continuous function of time t, we have $I_{V1}(T_1)=I_{V2}(0)$. Then

$$(P-(R_1+R_2))[1-e^{-\theta_V T_1}] = (R_1+R_2)[e^{\theta_V T_2}-1]$$

Using exponential series expansion and under the assumption $0<\theta<1$, we get

Using Misra (1975),

$$T_1 \approx \frac{(R_1+R_2)T_2\left[1+\frac{\theta_V T_2}{2}\right]}{(P-(R_1+R_2))} \tag{8}$$

Hence,

$$T = T_1 + T_2 = \frac{\left[P+\frac{\theta_V T_2}{2}(R_1+R_2)\right]T_2}{(P-(R_1+R_2))} \tag{9}$$

The cost components of vendor and buyers are as follows:

- Inventory holding cost IHC_V, of vendor per time unit is,

$$IHC_V = \frac{h_V}{T}\left[\int_0^{T_1} I_{V1}(t_1)dt_1 + \int_0^{T_2} I_{V2}(t_2)dt_2 - n_1\int_0^{\frac{T}{n_1}} I_{B1}(t)dt - n_2\int_0^{\frac{T}{n_2}} I_{B2}(t)dt\right] \tag{10}$$

$$(P-(R_1+R_2))T_1\left[1-\frac{\theta_V T_1}{2}\right] = (R_1+R_2)T_2\left[1+\frac{\theta_V T_2}{2}\right]$$

- Inventory holding costs IHC_{B1} for n_1-deliveries and IHC_{B2} for n_2-deliveries of two buyers per time unit are:

$$IHC_{B1} = \frac{n_1 h_{B1}}{T}\left[\int_0^{\frac{T}{n_1}} I_{B1}(t)dt\right] \quad \text{and} \quad IHC_{B2} = \frac{n_2 h_{B2}}{T}\left[\int_0^{\frac{T}{n_2}} I_{B2}(t)dt\right]$$

Hence,

$$IHC_B = IHC_{B1} + IHC_{B2} \tag{11}$$

The deteriorated cost for vendor per time unit is ,

$$DC_V = \frac{C_V}{T}\left[PT_1 - n_1 I_{mB1} - n_2 I_{mB2}\right] \tag{12}$$

and for buyers per time unit is

$$DC_B = \frac{n_1 C_B}{T}\left[I_{mB1} - \frac{R_1 T}{n_1}\right] + \frac{n_2 C_B}{T}\left[I_{mB2} - \frac{R_2 T}{n_2}\right] \tag{13}$$

The ordering cost of vendor per time unit is,

$$OC_V = \frac{A_V}{T} \tag{14}$$ and for buyers is,

$$OC_B = \frac{(n_1 + n_2)A_B}{T} \tag{15}$$

Hence, the buyer's total cost is

$$K_B = IHC_B + DC_B + OC_B \tag{16}$$

and for the vendor is

$$K_V = IHC_V + DC_V + OC_V \tag{17}$$

The integrated total cost of the inventory system is

$$K = K_B + K_V \tag{18}$$

Using (8) and (9) in (18), the integrated total cost of inventory system is function of continuous variable T_2 and discrete variables n_1 and n_2.

4. Solution Procedure:

To compute minimum integrated cost K we perform following steps:

- For fixed n_1 and n_2, equate derivatives of K with respect to T_2 to zero. Solve it for T_2 using mathematical software.
- The optimal values of n_i, i =1, 2 is obtained when the following condition is satisfied :

$$K(T_2(n_i - 1), n_i - 1) \geq K(T_2(n_i), n_i) \leq K(T_2(n_i + 1), n_i + 1) \tag{19}$$

- Compute optimal values of T_1 and T from (8) and (9) respectively.
- Compute the maximum inventory level of vendor I_{mV} and buyers I_{mB1} and I_{mB2} from, (5), (6) and (7) respectively.

5. Numerical example :

Consider the following parametric values in proper units :

$$[P, R_1, R_2, h_V, h_{B1}, h_{B2}, A_V, A_B, C_V, C_B, \theta_V, \theta_{B1}, \theta_{B2}]$$
$$= [2*10^6, 4*10^4, 8*10^4, 0.10, 0.15, 0.17, 5000, 200, 10, 12, 0.05, 0.08, 0.10]$$

Table 1 Optimal Solution of n_1 and n_2.

n_1	n_2	T_2	T_1	T	K_B	K_V	K
2	2	0.3602	0.0232	0.3834	16935.13	13412.43	30347.65
2	3	0.3812	0.0247	0.4079	14488.29	14986.34	29474.63
1	2	0.3722	0.0240	0.3961	21328.69	6925.20	28293.84
2	1	0.3222	0.0207	0.3430	24584.98	8282.90	32867.81
3	2	0.3645	0.0235	0.3879	14156.82	14870.27	31026.98
4	6	0.4258	0.0275	0.4533	11086.10	19846.60	30932.73
5	6	0.4298	0.0277	0.4576	11034.50	20488.80	31523.25

Vendor's point of view

Buyer's point of view

Integrated

If the buyers agree for integrated policy, the optimal deliveries are $n_1 = 2$ and $n_2 = 3$ instead of their original optimal deliveries $n_1 = 4$ and $n_2 = 6$, they will incur an increased cost of \$ 1458. The vendor will have cost savings of \$4860. The percentage of integrated cost reduction is 4.95%. Here vendor is beneficial. Thus, to develop beneficial situation for both the vendor and the buyers, the vendor should offer some discount of his savings due to the integrated approach for a long-run strategy.

Next, we carry out sensitivity analysis by changing model parameter in the range of -10%, -5%, 5% and 10%.

Table 2 : Sensitivity analysis when demand R_2 varies.

R_2	7.2	7.6	8	8.4	8.8
I_{mV}	44855	45647	46425	47188	47938
I_{mB1}	8224	8224	8224	8224	8224
I_{mB2}	10950	10950	10950	10950	10950
n_1*	2	2	2	2	2
N_2*	3	3	3	3	3
K*	28606	29045	29474	29894	30305
I_{mV}	49920	50793	51649	52491	53318
I_{mB1}	8224	8224	8224	8224	8224
I_{mB2}	10950	10950	10950	10950	10950
$n_1^{\#}$	4	4	4	4	4
$n_2^{\#}$	6	6	6	6	6
$K^{\#}$	30174	30477	30932	31546	31984
PICR%	5.48	4.93	4.95	5.53	5.54

Table 3 : Sensitivity analysis when production rate varies.

P	18	19	20	21	22
I_{mV}	46360	46394	46425	46452	46477
I_{mB1}	8224	8224	8224	8224	8224
I_{mB2}	10950	10950	10950	10950	10950
n_1*	2	2	2	2	2
n_2*	3	3	3	3	3
K*	29304	29397	29472	29548	29614
I_{mV}	51594	51623	51649	51672	51694
I_{mB1}	8224	8224	8224	8224	8224
I_{mB2}	10950	10950`	10950	10950	10950
$n_1^{\#}$	4	4	4	4	4
$n_2^{\#}$	6	6	6	6	6
$K^{\#}$	30742	30842	30932	31013	31088
PICR%	4.91	4.92	4.95	4.96	4.98

Table 4 : Sensitivity analysis when C_v changes.

C_V	8	9	10	11	12
I_{mV}	47148	46782	46425	46076	45735
I_{mB1}	8224	8224	8224	8224	8224
I_{mB2}	10950	10950	10950	10950	10950
n_1*	2	2	2	2	2
n_2*	3	3	3	3	3
K*	29061	29269	29474	29679	29882
I_{mV}	54280	52916	51649	50470	49367
I_{mB1}	8224	8224	8224	8224	8224
I_{mB2}	10950	10950`	10950	10950	10950
$n_1^{\#}$	4	4	4	4	4
$n_2^{\#}$	6	6	6	6	6
$K^{\#}$	29470	30210	30932	31638	32329
PICR%	1.41	3.22	4.95	6.60	8.19

Table 5 : Sensitivity analysis when C_B changes.

C_B	10	11	12	13	14
I_{mV}	49434	47859	46425	45111	43902
I_{mB1}	8224	8224	8224	8224	8224
I_{mB2}	10950	10950	10950	10950	10950
n_1*	2	2	2	2	2
n_2*	3	3	3	3	3
K*	27674	28588	29474	30334	31170
I_{mV}	53352	52480	51649	50856	50098
I_{mB1}	8224	8224	8224	8224	8224
I_{mB2}	10950	10950`	10950	10950	10950
$n_1^{\#}$	4	4	4	4	4
$n_2^{\#}$	6	6	6	6	6
$K^{\#}$	29948	30444	30932	31412	31885
PICR%	8.22	6.49	4.95	3.55	2.29

Table 6 : Sensitivity analysis when A_V changes.

A_V	4500	4750	5000	5250	5500
I_{mV}	44428	45437	46425	47393	48342
I_{mB1}	8224	8224	8224	8224	8224
I_{mB2}	10950	10950	10950	10950	10950
n_1*	2	2	2	2	2
n_2*	3	3	3	3	3
K*	28219	28853	29474	30082	30678
I_{mV}	49750	50708	51649	52574	53484
I_{mB1}	8224	8224	8224	8224	8224
I_{mB2}	10950	10950`	10950	10950	10950
$n_1^{\#}$	4	4	4	4	4
$n_2^{\#}$	6	6	6	6	6
$K^{\#}$	29807	30375	30932	31480	32018
PICR%	5.62	5.28	4.95	4.65	4.37

Table 7 : Sensitivity analysis when h_{B2} changes.

h_{B2}	0.15	0.16	0.17	0.18	0.19
I_{mV}	46597	46511	46425	46339	46254
I_{mB1}	8224	8224	8224	8224	8224
I_{mB2}	10950	10950	10950	10950	10950
n_1*	2	2	2	2	2
n_2*	3	3	3	3	3
K*	29365	29420	29474	29529	29583
I_{mV}	51751	51700	51649	51599	51548
I_{mB1}	8224	8224	8224	8224	8224
I_{mB2}	10950	10950`	10950	10950	10950
$n_1^{\#}$	4	4	4	4	4
$n_2^{\#}$	6	6	6	6	6
$K^{\#}$	30872	30902	30932	30962	30992
PICR%	5.13	5.04	4.95	4.85	4.76

Table 8 : Sensitivity analysis when Θ_V changes.

θ_V	0.03	0.04	0.05	0.06	0.07
I_{mV}	55088	50198	46425	43400	40904
I_{mB1}	8224	8224	8224	8224	8224
I_{mB2}	10950	10950	10950	10950	10950
n_1*	2	2	2	2	2
n_2*	3	3	3	3	3
K*	24789	27233	29474	31556	33508
I_{mV}	62031	56124	51649	48108	45215
I_{mB1}	8224	8224	8224	8224	8224
I_{mB2}	10950	10950`	10950	10950	10950
$n_1^{\#}$	4	4	4	4	4
$n_2^{\#}$	6	6	6	6	6
$K^{\#}$	25694	28434	30932	33242	35401
PICR%	3.65	4.40	4.95	5.34	5.65

The observations drawn from the senstivity analysis are as follows :

1. The PICR is more sensitive to C_V, C_B, A_V and less sensitive to R_2, P, h_{B2}, and
2. When the market demand R2 increases, the optimal maximum inventory increases.
3. Increase in production rate decreases maximum inventories of two buyers.
4. Increase in inventory holding cost of buyer 2, h_{B2} decreases the maximum inventory level of vendor.

6. Conclusion:

In this paper, a production inventory policy for different deterioration rates of vendor and two buyers is developed. It is assumed that demand rate of both the buyers are different. The sensitivity analysis is carried out suggests that the integrated approach reduces the total joint cost for both the players. The incentives in the form of quantity discount offered by the vendor may be turn out to be a more realistic approach for the research.

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